

Problem 11188. *Proposed by David H. Bailey, Lawrence Berkeley Lab, Berkeley, CA; Jonathan M. Borwein, Dalhousie University, Nova Scotia, Canada; and David M. Bradley, University of Maine, Orono, Maine.*

Let n be a positive integer. Evaluate in closed form the generalized hypergeometric series

$${}_3F_2\left(\begin{matrix} 3n, n+1, -n \\ 2n+1, n+1/2 \end{matrix} \middle| \frac{1}{4}\right) = \sum_{k=0}^n \frac{(3n)_k (n+1)_k (-n)_k}{(2n+1)_k (n+1/2)_k} \cdot \frac{(1/4)^k}{k!}.$$

Here for non-negative integers k and any a ,

$$(a)_k := \prod_{j=1}^k (a+j-1) = \frac{\Gamma(a+k)}{\Gamma(a)}$$

is the usual Pochhammer symbol. The identity is key to proving an infinite class of “Apéry-like” series acceleration formulae for values of the Riemann zeta function at even positive integers: see our preprint <http://www.arXiv.org/math.NT/0505270>