



Counting Ordered Pairs

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6. L. Pudwell, *Enumeration Schemes for Pattern-Avoiding Words and Permutations*, Ph.D. thesis, Rutgers University, 2008.
7. R. Simion and F. W. Schmidt, Restricted permutations, *European J. Combin.* **6** (1985) 383–406.

Summary This paper explores a surprising connection between a geometry problem and a result in enumerative combinatorics. First, we find the surface areas of certain solids formed from unit cubes. Next, we enumerate multiset permutations which avoid the patterns {132, 231, 2134}. Finally, we give a bijection between the faces of the solids and the set of permutations.

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After Cantor [3, p. 107] (cf. also [2]), the standard method of enumerating the set $\mathbf{Z}^+ \times \mathbf{Z}^+$ of ordered pairs of positive integers is to list the entries by traversing successive diagonals, beginning with (1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1), and so on. An explicit bijection that accomplishes this is $\varphi : \mathbf{Z}^+ \times \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$ defined by

$$\varphi(m, n) = m + \frac{(m+n-1)(m+n-2)}{2}.$$

Providing an algebraic proof that φ is indeed a bijection is an instructive exercise.

By exploiting the multiplicative structure of the codomain, we can construct a map $\psi : \mathbf{Z}^+ \times \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$ which is immediately recognized as a bijection. (No need to resort to algebraic calculation or a pictorial argument with diagonals.) For each pair of positive integers m and n , let $\psi(m, n) = 2^{m-1}(2n-1)$. Bijectivity of ψ is equivalent to the fact that every positive integer has a unique representation as the product of an odd positive integer and a non-negative integer power of 2. As one referee noted, this fact is also key to Glaisher's bijection between partitions of a positive integer into odd parts and partitions with distinct parts [1, Ex. 2.2.6; 4, p. 12].

REFERENCES

1. David Bressoud, *Proofs and Confirmations: The Story of the Alternating Sign Matrix Conjecture*, MAA and Cambridge University Press, 1999.
2. Georg Cantor, Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen, *J. Reine Angew. Math.* **77** (1874) 258–261. doi:10.1515/crll.1874.77.258
3. Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*, trans. by Philip E. B. Jourdain, Open Court Publishing, La Salle, Illinois, 1941.
4. Percy A. MacMahon, *Combinatory Analysis*, Vol. II, Chelsea Publishing, New York, 1984.