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Summary This paper explores a surprising connection between a geometry problem and a result in enumerative combinatorics. First, we find the surface areas of certain solids formed from unit cubes. Next, we enumerate multiset permutations which avoid the patterns {132, 231, 2134}. Finally, we give a bijection between the faces of the solids and the set of permutations.

Counting Ordered Pairs

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After Cantor [**3**, p. 107] (cf. also [**2**]), the standard method of enumerating the set $\mathbf{Z}^+ \times \mathbf{Z}^+$ of ordered pairs of positive integers is to list the entries by traversing successive diagonals, beginning with (1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1), and so on. An explicit bijection that accomplishes this is $\varphi : \mathbf{Z}^+ \times \mathbf{Z}^+ \to \mathbf{Z}^+$ defined by

$$\varphi(m, n) = m + \frac{(m+n-1)(m+n-2)}{2}.$$

Providing an algebraic proof that φ is indeed a bijection is an instructive exercise.

By exploiting the multiplicative structure of the codomain, we can construct a map $\psi : \mathbf{Z}^+ \times \mathbf{Z}^+ \to \mathbf{Z}^+$ which is immediately recognized as a bijection. (No need to resort to algebraic calculation or a pictorial argument with diagonals.) For each pair of positive integers m and n, let $\psi(m, n) = 2^{m-1}(2n-1)$. Bijectivity of ψ is equivalent to the fact that every positive integer has a unique representation as the product of an odd positive integer and a non-negative integer power of 2. As one referee noted, this fact is also key to Glaisher's bijection between partitions of a positive integer into odd parts and partitions with distinct parts [**1**, Ex. 2.2.6; **4**, p. 12].

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